Engineering Notes

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Hypersonic Fin Aerodynamics

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Introduction

IT is known that in hypersonic flows the pressure coefficient varies nonlinearly with thickness ratio and angle of incidence, in contrast to the linear laws for thin two-dimensional bodies at moderate supersonic speeds. Therefore, effects of the dihedral angle, cant angle, angle of attack, rolling rate, and shape of fin on the aerodynamic properties cannot be separately treated as in the linear supersonic flow theory when the flow is within the hypersonic range.

Using a shock-expansion theory,[1] an attempt is made in this Note to obtain the expressions for various aerodynamic coefficients of a fin with the combined effects of the dihedral angle, cant angle, angle of attack, rolling rate, and the shape of the fin when the flow is hypersonic.

Apparent Velocity and Effective Angle of Attack

Construct a coordinate system as shown in Fig. 1. X is along the body axis. Y is so defined that XYZ forms a right-hand system. Z is perpendicular to X and is in the plane of X and V. V is the freestream velocity making an angle of attack α to the X axis. V is located in the XZ plane.

Introduce a specific sequence of coordinate transformations by first rotating the XYZ system by the dihedral angle Γ counterclockwise about the X axis. The resultant coordinate system is labeled $\bar{X}\bar{Y}\bar{Z}$. The $\bar{Y}$ axis is thus along the spanwise direction of a fin. The $\bar{X}\bar{Y}\bar{Z}$ system is then rotated about the $\bar{Y}$ axis counterclockwise by the cant angle δ to obtain the desired $X'Y'Z'$ system of axes in which the shock-expansion theory can be conveniently applied. It should be emphasized that the specific order of rotation is critical. This is to ensure the correspondence to the fin configuration in flight. The relation between the XYZ system and the $X'Y'Z'$ system is given by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = [A] \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}$$

where

$$[A] = \begin{bmatrix} \cos\delta & 0 & \sin\delta \\ \sin\Gamma\sin\delta & \cos\Gamma & -\sin\Gamma\cos\delta \\ -\cos\Gamma\sin\delta & \sin\Gamma & \cos\Gamma\cos\delta \end{bmatrix} \tag{1}$$

When the vehicle has a roll angular velocity about the X axis, $\boldsymbol{P}$, the apparent velocity V_a at a wing section, is defined by $V_a = V_a - \boldsymbol{P} \times \boldsymbol{r}$, where $\boldsymbol{r} = x'\boldsymbol{i}' + y'\boldsymbol{j}'$ is the coordinates of a point of the fin planform ($Z' = 0$) and $\boldsymbol{P} = -P\boldsymbol{i}$ is the roll angular velocity of the vehicle. Thus, the apparent velocity V_a has the following components in the $X'Y'Z'$ system.

$$V \begin{pmatrix} \cos\alpha\cos\delta - \sin\alpha\sin\delta\cos\Gamma - Py'\sin\delta/V \\ \sin\alpha\sin\Gamma + Px'\sin\delta/V \\ \cos\alpha\sin\delta + \sin\alpha\cos\delta\cos\Gamma + Py'\cos\delta/V \end{pmatrix} \tag{2}$$

Therefore the magnitude of the effective angle of attack α_e to the fin surface with a unit normal to the surface $\boldsymbol{n}$ is given by

$$\sin\alpha_e = \frac{|V_a \cdot \boldsymbol{n}|}{|V_a|}, \qquad 0 \le \alpha_e \le \pi/2 \tag{3}$$

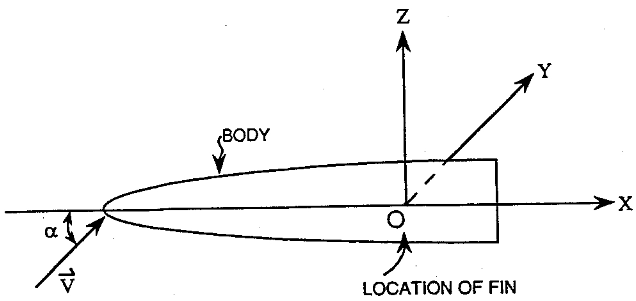

Fig. 1 Coordinate system.

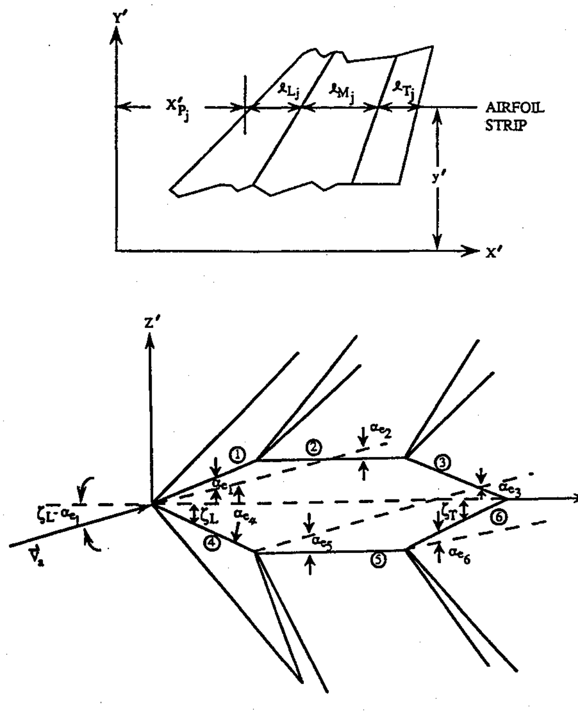

Fig. 2 Modified double-wedge fin.

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Hence, the effective angle of attack is α_e if $V_a \cdot n$ is positive and $-\alpha_e$ if $V_a \cdot n$ is negative. Consider the configuration of a modified double-wedge fin as shown in Fig. 2. The normal to the fin surface is assumed to lie in the $X'Z'$ plane. The magnitude of the effective angle of attack α_{ei} to the ith surface region of a fin, $i = 1, 2, \ldots, 6$, is therefore determined.

$$\sin \alpha_{e1} = \frac{-V}{|V_a|}\left[\begin{array}{l} \sin \zeta_L(\sin \alpha \sin \delta \cos \Gamma - \cos \alpha \cos \delta) + \dfrac{Py'}{V} \sin \zeta_L \sin \delta \\ + \cos \zeta_L(\cos \alpha \sin \delta + \sin \alpha \cos \delta \cos \Gamma) + \dfrac{Py'}{V} \cos \zeta_L \cos \delta \end{array}\right] \tag{4}$$

$$\sin \alpha_{e2} = \frac{V}{|V_a|}\left(\cos \alpha \sin \delta + \sin \alpha \cos \delta \cos \Gamma + \frac{Py'}{V} \cos \delta\right) \tag{5}$$

$$\sin \alpha_{e3} = \frac{V}{|V_a|}\left[\begin{array}{l} \sin \zeta_T(\cos \alpha \cos \delta - \sin \alpha \sin \delta \cos \Gamma) - \dfrac{Py'}{V} \sin \zeta_T \sin \delta \\ + \cos \zeta_T(\cos \alpha \sin \delta + \sin \alpha \cos \delta \cos \Gamma) + \dfrac{Py'}{V} \cos \zeta_T \cos \delta \end{array}\right] \tag{6}$$

$$\sin \alpha_{e4} = \frac{-V}{|V_a|}\left[\begin{array}{l} \sin \zeta_L(\sin \alpha \sin \delta \cos \Gamma - \cos \alpha \cos \delta) + \dfrac{Py'}{V} \sin \zeta_L \sin \delta \\ - \cos \zeta_L(\cos \alpha \sin \delta + \sin \alpha \cos \delta \cos \Gamma) - \dfrac{Py'}{V} \cos \zeta_L \cos \delta \end{array}\right] \tag{7}$$

$$\sin \alpha_{e5} = \frac{V}{|V_a|}\left(\cos \alpha \sin \delta + \sin \alpha \cos \delta \cos \Gamma + \frac{Py'}{V} \cos \delta\right) \tag{8}$$

$$\sin \alpha_{e6} = \frac{V}{|V_a|}\left[\begin{array}{l} \sin \zeta_T(\cos \alpha \cos \delta - \sin \alpha \sin \delta \cos \Gamma) - \dfrac{Py'}{V} \sin \zeta_T \sin \delta \\ - \cos \zeta_T(\cos \alpha \sin \delta + \sin \alpha \cos \delta \cos \Gamma) - \dfrac{Py'}{V} \cos \zeta_T \cos \delta \end{array}\right] \tag{9}$$

where $0 \le \alpha_{ei} \le \pi/2$, $i = 1, 2, \ldots, 6$; and ζ_L and ζ_T are leading- and trailing-edge wedge half-angle, respectively.

Shock-Expansion Theory

In what follows, the shock-expansion theory[1] is applied to a two-dimensional airfoil strip with the following assumptions: the flow is hypersonic; the airfoil has a sharp leading edge; the flow behind the leading-edge shock is the same as an isentropic Prandtl-Meyer expansion, with only a single family of characteristics taken into account; the reflections of the Mach waves from the curved shock are weak and the reflections of the Mach waves from the streamlines are also weak; the fin-tip Mach cone correction is not considered; and flows in regions 1, 2, and 3 and in regions 4, 5, and 6 are independent of each other (see Fig. 2).

It has been shown by Eggers et al.[1] that the pressure coefficient at the surface of an airfoil satisfying the preceding assumptions has the expression

$$C_{pi} = \frac{2}{\gamma M_a^2}\left\{g(K)\left[1 - f(K)\left(1 - \frac{\theta_i}{\bar\theta}\right)\right]^{\frac{2\gamma}{\gamma-1}} - 1\right\} \tag{10}$$

where

$$g(K) = \frac{2\gamma}{\gamma+1}K_s^2 - \frac{\gamma-1}{\gamma+1}$$

$$f(K) = \frac{(\gamma - 1/2)K}{\sqrt{\left(\dfrac{2\gamma}{\gamma+1}K_s^2 - \dfrac{\gamma-1}{\gamma+1}\right)\left(\dfrac{\gamma-1}{\gamma+1} + \dfrac{2}{\gamma+1}\dfrac{1}{K_s^2}\right)}}$$

$$K_s = \frac{\gamma+1}{4}K + \sqrt{\left(\frac{\gamma+1}{4}K\right)^2 + 1}$$

and where γ is the ratio of specific heats, $K = M_a\bar\theta$ is the hypersonic similarity parameter, $M_a = V_a/a_\infty$ is the apparent Mach number for the flow, and $\bar\theta$ is the flow deflection angle at the leading edge of the airfoil. Thus, $\bar\theta = \alpha_{e1}$ in the upper surface and $\bar\theta = \alpha_{e4}$ in the lower surface. θ_i is the local angle of incidence of the ith region.

$$\theta_1 = \alpha_{e1}, \theta_2 = -\alpha_{e2}, \theta_3 = -\alpha_{e3}, \theta_4 = \alpha_{e4}, \theta_5 = \alpha_{e5}, \text{and } \theta_6 = -\alpha_{e6}$$

It should be noted that the slender-airfoil expression, Eq. (10), satisfies the hypersonic similarity law first deduced by Tsien.[2,3]

Aerodynamic Characteristics of a Fin

Applying a strip theory to a fin in a manner similar to the procedure given by Barrowman,[4] the normal force and axial force acting on the jth airfoil strip are

$$F'_{Nj} = \sum_{i=1}^{3}(C_{pi+3} - C_{pi})A_i$$

$$F'_{Aj} = (C_{p1} + C_{p4})A_1 \tan \zeta_L - (C_{p3} + C_{p6})A_3 \tan \zeta_T$$

where C_{pi} the local pressure coefficient, A_i is the local planform region area in Fig. 2, and $i = 1, 2, \ldots, 6$. Thus, the lift force and drag force on the jth strip are then

$$F'_{Lj} = F'_{Nj}\cos(\zeta_L - \theta_1) - F'_{Aj}\sin(\zeta_L - \theta_1)$$

$$F'_{Dj} = F'_{Nj}\sin(\zeta_L - \theta_1) + F'_{Aj}\cos(\zeta_L - \theta_1)$$

The center of pressure of each region in a strip is located at the centroid of the surface constituting the region. Since the pressure in such a region is uniform, the center of pressure of the jth strip is given by

$$\bar{x}_j' = \left[(C_{p4} - C_{p1})\ell_{Lj}\left(x_{pj}' + \frac{\ell_{Lj}}{2}\right) + (C_{p5} - C_{p2})\ell_{Mj}\left(x_{pj}' + \ell_{Lj} + \frac{\ell_{Mj}}{2}\right) + (C_{p6} - C_{p3})\ell_{Tj}\left(x_{pj}' + \ell_{Lj} + \ell_{Mj} + \frac{\ell_{Tj}}{2}\right)\right] / [(C_{p4} - C_{p1})\ell_{Lj} + (C_{p5} - C_{p2})\ell_{Mj} + (C_{p6} - C_{p3})\ell_{Tj}]$$

where ℓ_{Lj}, ℓ_{Mj}, and ℓ_{Tj} are local airfoil-strip chords defined in Fig. 2. Hence, the strip pitching moment about the Y' axis, the strip rolling moment about the X' axis, and the strip yawing moment about the Z' axis are, respectively,

$$M_{pj}' = -F_{Nj}'\bar{x}_j', \qquad M_{Rj}' = F_{Nj}'y_j', \qquad M_{yj}' = -F_{Aj}'y_j'$$

Summing j over all of the strips of a fin, one obtains for a fin

$$F_N' = \sum_{j=1}^{n} F_{Nj}', \qquad F_L' = \sum_{j=1}^{n} F_{Lj}'$$

$$F_A' = \sum_{j=1}^{n} F_{Aj}', \qquad F_D' = \sum_{j=1}^{n} F_{Dj}'$$

$$M_p' = \sum_{j=1}^{n} M_{pj}', \qquad M_R' = \sum_{j=1}^{n} M_{Rj}', \qquad M_Y' = \sum_{j=1}^{n} M_{Yj}'$$

The center-of-pressure coordinates are then

$$\bar{X}' = M_p'/F_N', \qquad \bar{Y}' = M_R'/F_N'$$

Transforming back to the XYZ system by Eq. (1), the force and the moment on a fin have, respectively, the components

$$\begin{pmatrix} F_A \\ F_S \\ F_N \end{pmatrix} = [A] \begin{pmatrix} F_A' \\ 0 \\ F_N' \end{pmatrix} = \begin{bmatrix} F_N'\sin\delta + F_A'\cos\delta \\ \sin\Gamma\,(F_A'\sin\delta - F_N'\cos\delta) \\ \cos\Gamma\,(F_N'\cos\delta - F_A'\sin\delta) \end{bmatrix}$$

$$\begin{pmatrix} M_R \\ M_P \\ M_Y \end{pmatrix} = [A] \begin{pmatrix} M_R' \\ M_P' \\ M_Y' \end{pmatrix} = \begin{bmatrix} M_R'\cos\delta + M_Y'\sin\delta \\ M_P'\cos\Gamma + \sin\Gamma\,(M_R'\sin\delta - M_Y'\cos\delta) \\ M_P'\sin\Gamma + \cos\Gamma\,(M_Y'\cos\delta - M_Y'\sin\delta) \end{bmatrix}$$

Obviously, the fin dihedral angle Γ causes a side force F_s to the vehicle. Note that F_A' and F_N' are also Γ-dependent. Thus, the task of minimizing or eliminating F_s is very much involved in hypersonic flow conditions. Furthermore, note that in hypersonic flows it is difficult to decouple the roll forcing moment and the roll damping moment, or the pitch forcing moment and the pitch damping moment, in general.

The associated aerodynamic coefficients are, respectively,

Lift coefficient:

$$C_N = (F_N\cos\alpha - F\sin\alpha)/A_r$$

Wave drag coefficient:

$$C_{DW} = (F_N\sin\alpha + F_A\cos\alpha)/A_r$$

Pitching moment coefficient:

$$C_m = M_p/A_rL_r$$

Rolling moment coefficient:

$$C_\ell = M_R/A_rL_r$$

where A_r is the reference area and L_r is the reference length.

The preceding analysis is for one fin panel. The aerodynamic characteristics for multiple fin panels can be obtained by summing corresponding aerodynamic characteristics over all fins.

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Columbus: To Mars with Solar-Electric Propulsion

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Introduction

HUMAN travel to Mars is being considered as part of the space exploration initiative. Nuclear propulsion appears to have several benefits, but the problem of launching concentrated fissionable material into orbit and starting a reactor in a low-Earth orbit may prove too difficult to be solved in a society that is increasingly concerned with safety and ecology. Chemical propulsion leads to very massive vehicles when the desired mission design includes a rapid transfer between planets to minimize radiation damage and also includes a long stay at Mars to allow the crew to recover from the flight out before the return flight. Solar-electric propulsion offers a reasonable alternative, but it also has problems. Assembling a large array at a low-Earth-orbit node is difficult, and the long trip time from the assembly orbit to the high-Earth orbit where the crew would join the main vehicle leads to awkward mission planning.

Contents

A design for a human trip to Mars using solar-electric propulsion is proposed. The key feature of this design is that the solar array is divided into three identical parts. Each part carries cargo to a rendezvous point. Because each of the three parts carries a different cargo mass, they can be launched from the low-Earth-orbit assembly point at different times and all arrive at the rendezvous point at the same time.

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